Power and Smith Chart

Power Flow

- How much power is flowing and reflected?
  - Instantaneous P(d,t) = v(d,t).i(d,t)
    - Incident
    - Reflected

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$

  - Average power: $Pav = Pav^i + Pav^r$
    - Time-domain Approach
    - Phasor-domain Approach (z and t independent)
      - ½ Re{$I^*(z)$. V(z)}

Instantaneous Power Flow

$$\begin{aligned} \upsilon(d,t) &= \mathfrak{Re}[\widetilde{V} e^{j\omega t}] \\ &= \mathfrak{Re}[|V_0^+|e^{j\phi^+}(e^{j\beta d} + |\Gamma|e^{j\theta_r}e^{-j\beta d})e^{j\omega t}] \\ &= |V_0^+|[\cos(\omega t + \beta d + \phi^+) \\ &\qquad + |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)], \end{aligned} \quad (2.99a)$$

$$\begin{aligned} i(d,t) &= \frac{|V_0^+|}{Z_0}[\cos(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)], \end{aligned} \quad (2.99b)$$

$$\begin{aligned} P(d,t) &= \upsilon(d,t)\, i(d,t) \\ &= |V_0^+|[\cos(\omega t + \beta d + \phi^+) \\ &\qquad + |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)] \\ &\quad \times \frac{|V_0^+|}{Z_0}[\cos(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma|\cos(\omega t - \beta d + \phi^+ + \theta_r)] \\ &= \frac{|V_0^+|^2}{Z_0}[\cos^2(\omega t + \beta d + \phi^+) \\ &\qquad - |\Gamma|^2\cos^2(\omega t - \beta d + \phi^+ + \theta_r)] \end{aligned}$$

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{Z_0}\cos^2(\omega t + \beta d + \phi^+) \qquad \text{(W)},$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2\frac{|V_0^+|^2}{Z_0}\cos^2(\omega t - \beta d + \phi^+ + \theta_r)$$

Using the trigonometric identity

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x),$$

the expressions in Eq. (2.101) can be rewritten as

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$\begin{aligned} P^{\mathrm{r}}(d,t) = -|\Gamma|^2\frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d \\ + 2\phi^+ + 2\theta_r)]. \end{aligned}$$

The power oscillates at twice the rate of the voltage or current.

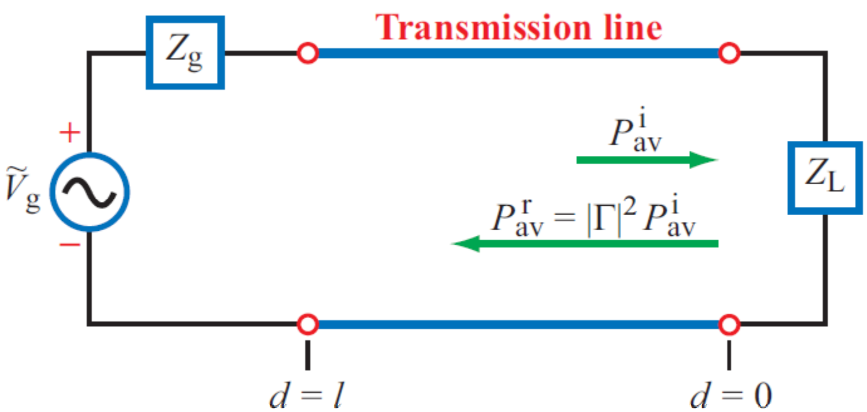
Average Power

(Phasor Approach)

Avg Power: ½ Re{I(z) * V_(z)}

$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)]$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$



Fraction of power reflected!

$$P^{\mathrm{i}}_{\mathrm{av}} = \frac{|V_0^+|^2}{2Z_0} \quad \text{(W)}, \qquad (2.104)$$

which is identical with the dc term of $P^{\mathrm{i}}(d,t)$ given by Eq. (2.102a). A similar treatment for the reflected wave gives

$$P^{\mathrm{r}}_{\mathrm{av}} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P^{\mathrm{i}}_{\mathrm{av}}. \qquad (2.105)$$

The average reflected power is equal to the average incident power, diminished by a multiplicative factor of $|\Gamma|^2$.

Example

- Assume Zo=50 ohm, ZL=100+i50 ohm; What fraction of power is reflected?

$$P_{av}^{r} = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P_{av}^{i}. \quad (2.105)$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

```
>> x=(50+i*50)/(150+i*50)

x =

   0.4000 + 0.2000i

>> mag=abs(x)

mag =

    0.4472

>> angle=cart2pol(.4,.2)

angle =

    0.4636
```

```
angle =

    0.4636

>> radtodeg(.4636)

ans =

   26.5623
```

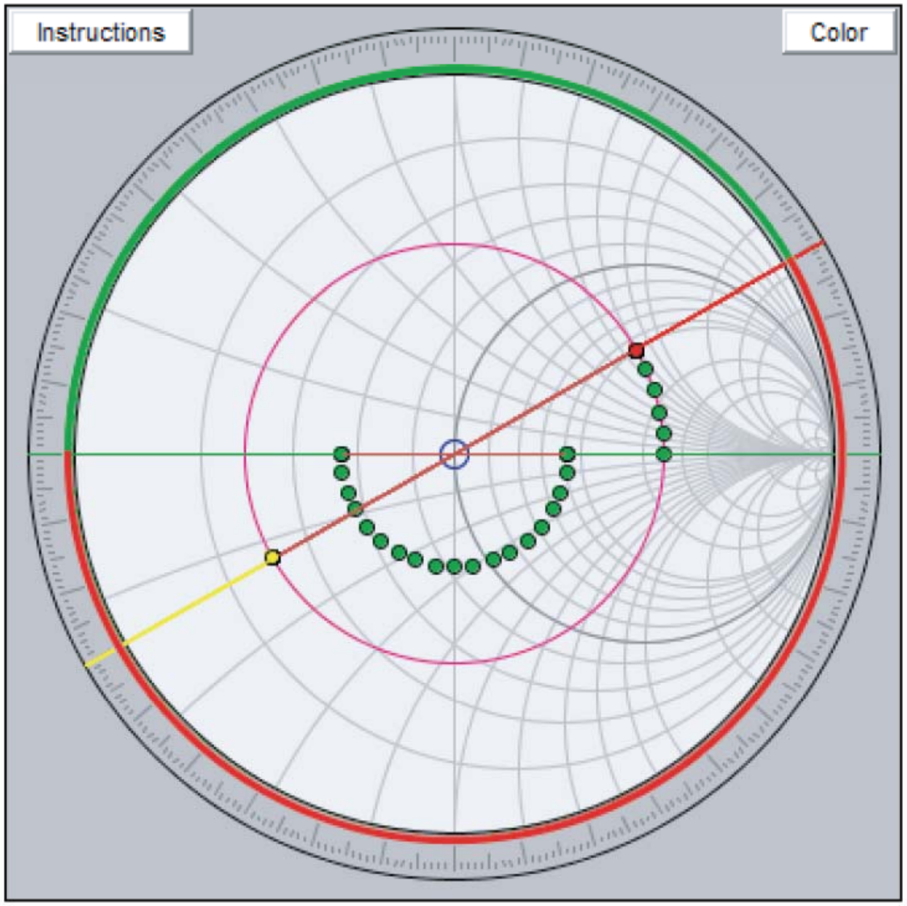
```
>> mag^2

ans =

    0.2000
```

20 percent!

Notes

The Smith Chart

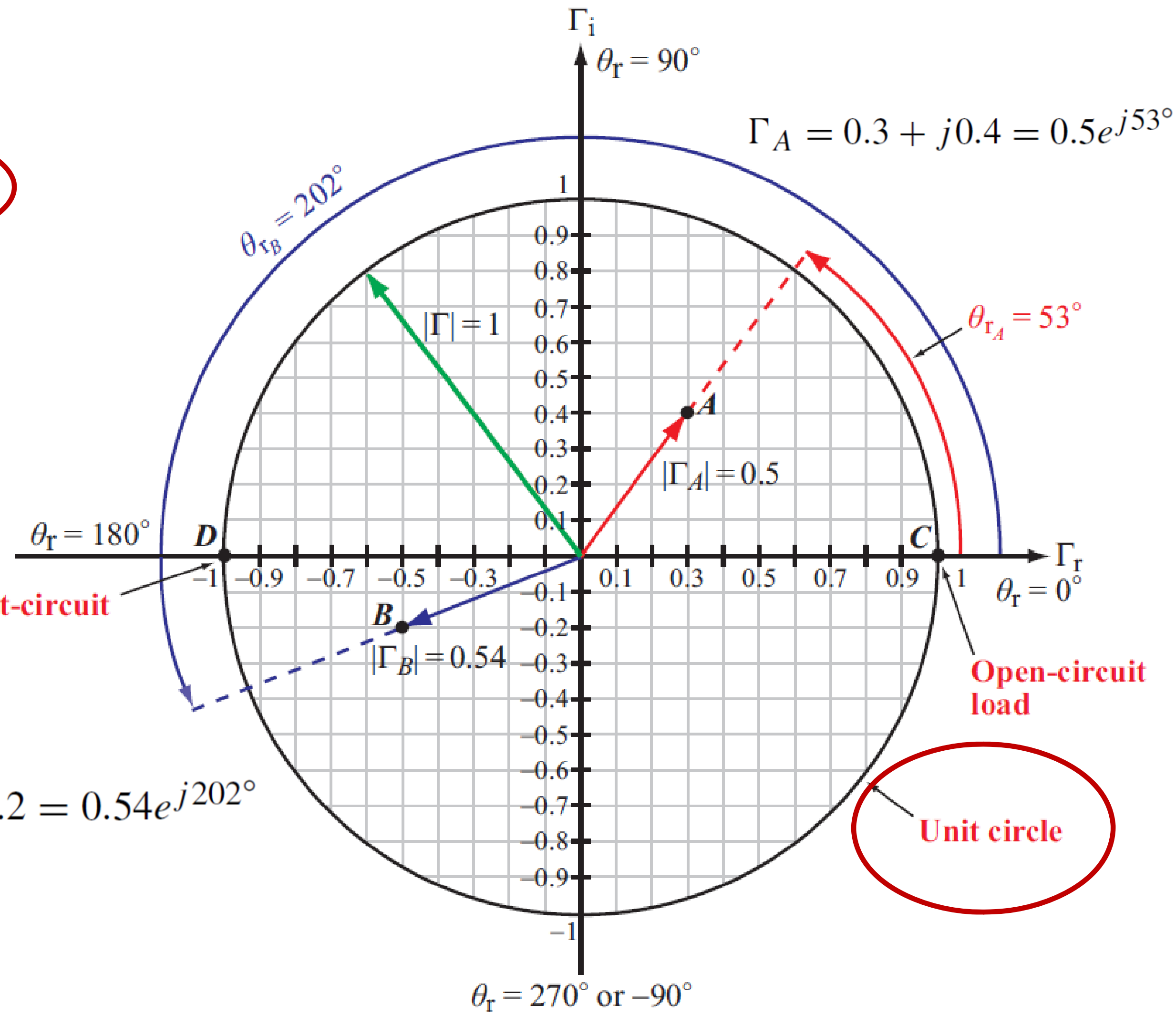
- Developed in 1939 by P. W. Smith as a graphical tool to analyze and design transmission-line circuits
- Today, it is used to characterize the performance of microwave circuits



Complex Plane

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$\Gamma_B = -0.5 - j0.2 = 0.54 e^{j202^\circ}$$



Smith Chart Parametric Equations

$$\Gamma = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

$$z_L = \frac{1+\Gamma}{1-\Gamma}. \qquad (2.112)$$

$$z_L = r_L + jx_L.$$

$$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$\Gamma = |\Gamma|e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Equation for a circle

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

For a given Coef. Of Reflection
various load combinations can be considered.
These combinations can be represented by
different circuits!
Smith Chart help us see these variations!

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

Smith Chart Parametric Equations

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2. \qquad (2.116)$$

r_L circles

r_L circles are contained inside the unit circle

Each node on the chart will tell us about the load characteristics and coef. of ref. of the line!

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2, \qquad (2.118)$$

x_L circles

Only parts of the x_L circles are contained within the unit circle

$$z_L = r_L + jx_L.$$

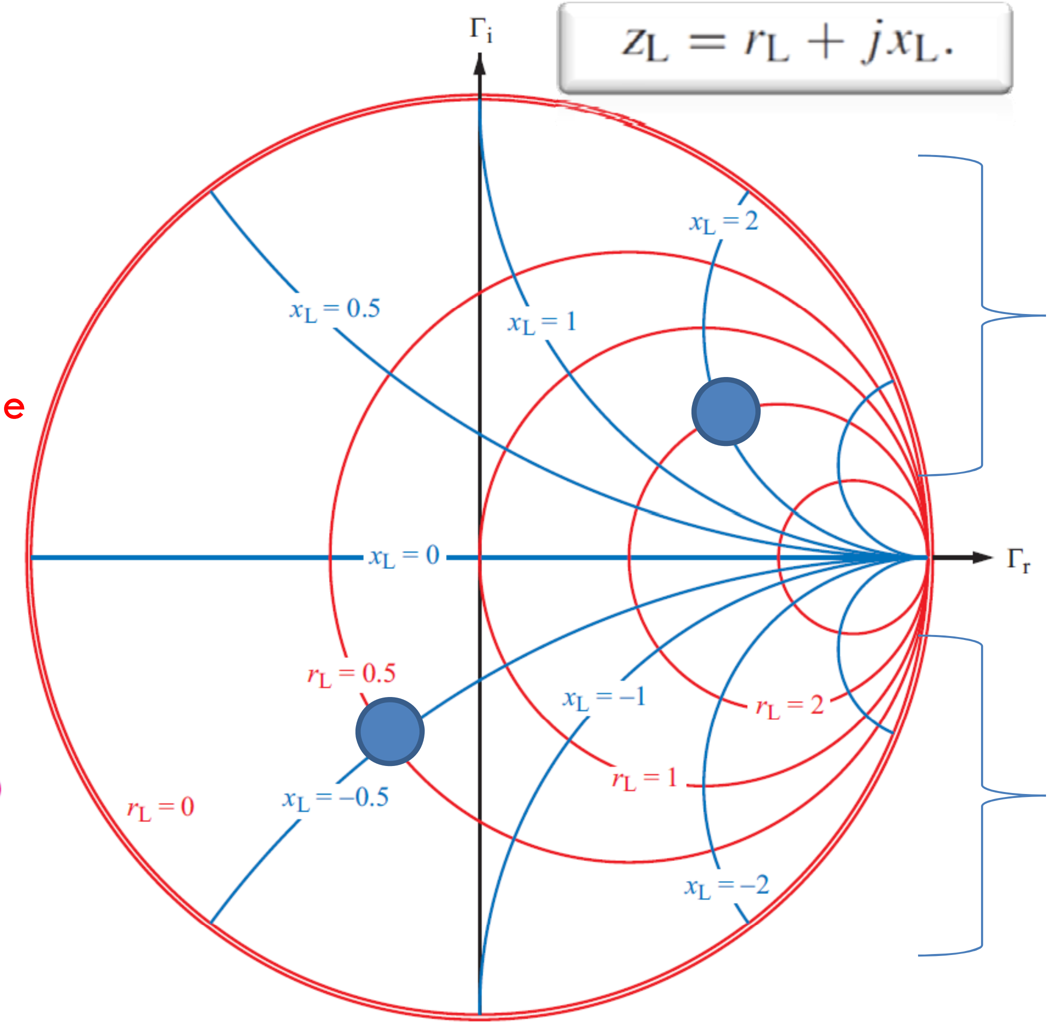
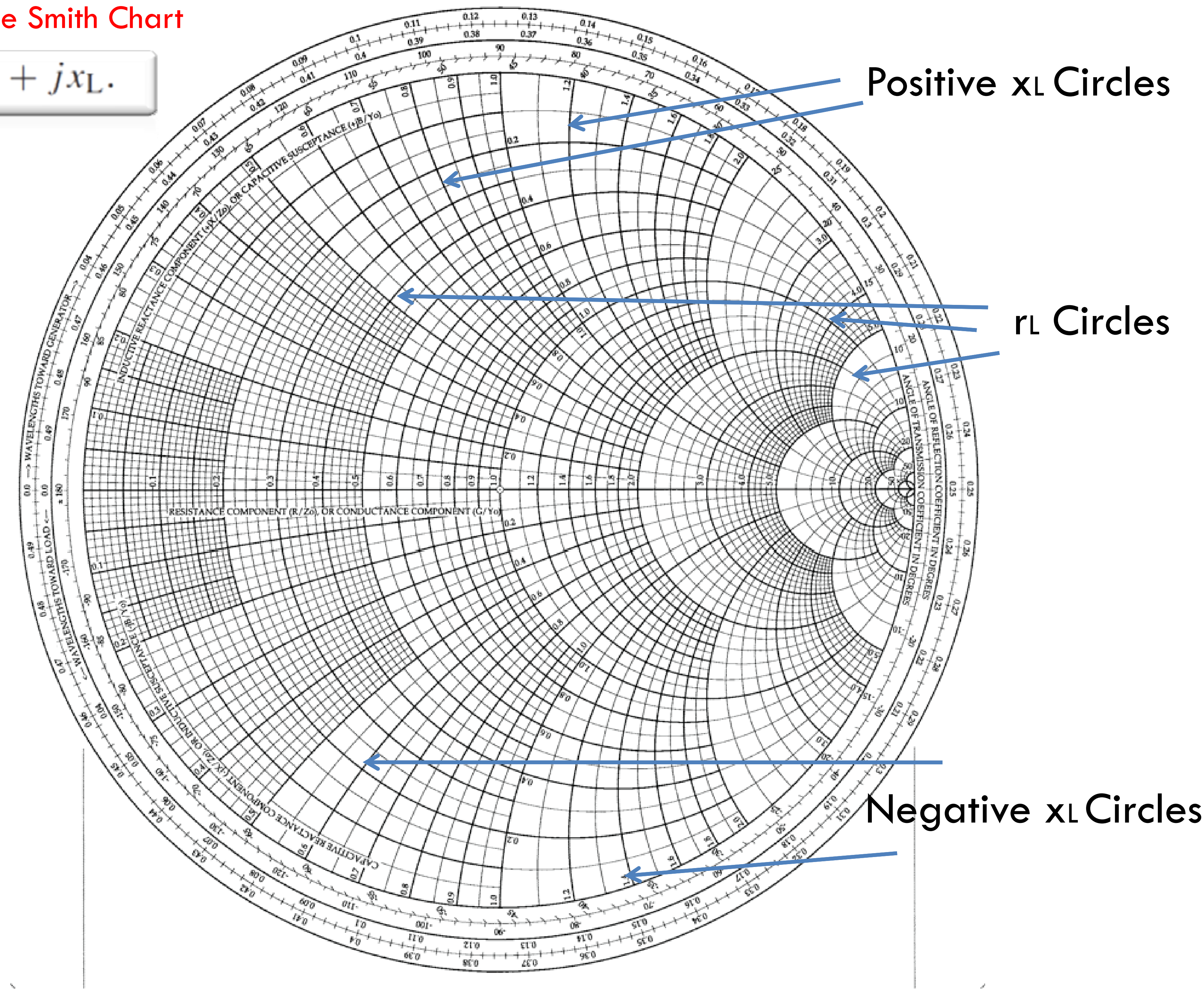


Figure 2-25: Families of r_L and x_L circles within the domain $|\Gamma| \leq 1$.

Complete Smith Chart

$$z_L = r_L + jx_L.$$



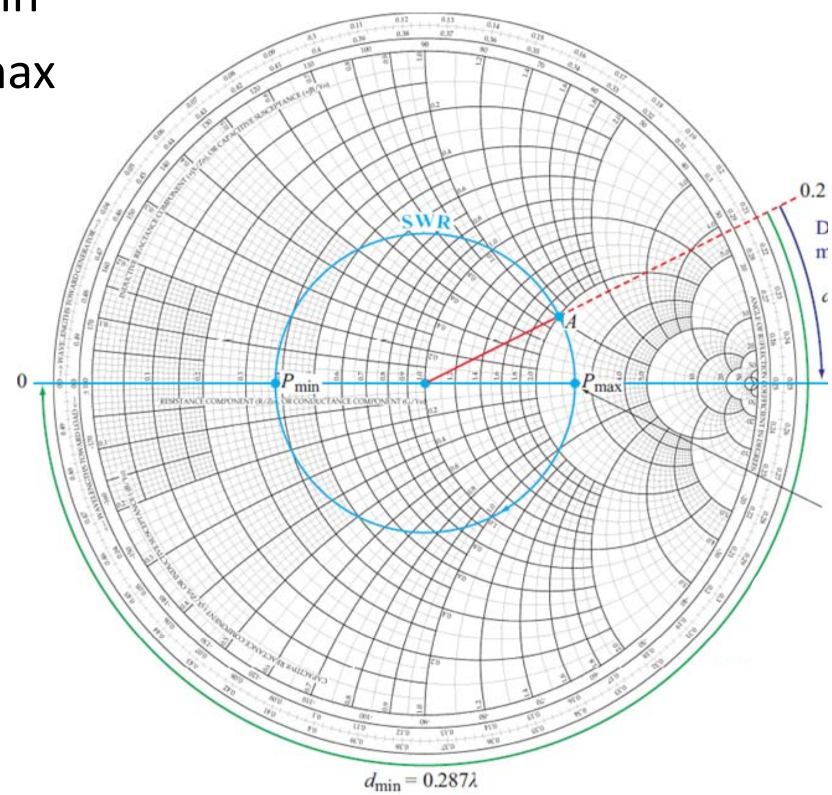
Basic Rules

- Given zL=ZL/Zo find the coefficient of reflection (COR)
  - Find zL on the chart (Pt. A)
  - Extend it and find the angle of COR - ANGLE OF REFLECTION COEFFICIENT
  - Use ruler to *measure* the magnitude of COR: OA/OP
- Find VSWR
  - Draw a circle with radius of ZL
- Find Zin
  - Find zL on the chart (Pt. A)
  - Extend it and find the angle of COR - ANGLE OF REFLECTION COEFFICIENT (pt. B)
  - Draw the SWR circle
  - Determine how far the load is from the generator: d (e.g., $d=3.3\lambda \rightarrow d=0.3\lambda$)
  - From Pt. B move clockwise by 0.3λ. (pt. C) on WTG
  - Draw a line from pt C. to the origin: OC
- Input impedance Zd (any point d)
  - Same as above – except $d=y\lambda$
- Find Yin
  - Find Zin
  - Extend the OC line to the opposite side of the chart OC`
  - The intersection of line OC` and SWR circle is Yin

Note
s

Basic Rules

- Find Z(dmin) or dmax
  - Find zL on the chart (Pt. A)
  - Extend it and find the angle of COR - ANGLE OF REFLECTION COEFFICIENT (pt. B)
  - From pt. A to Vmin with be the dmin
  - From pt. A to Vmax with be the dmax



Note
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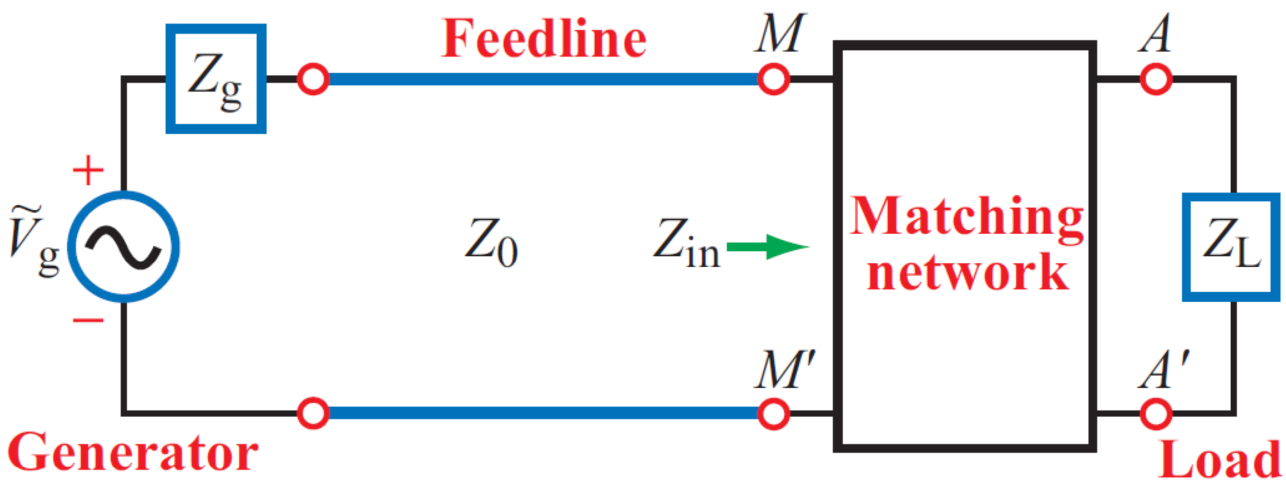
Refer to
the examples in your notes

Matching Network



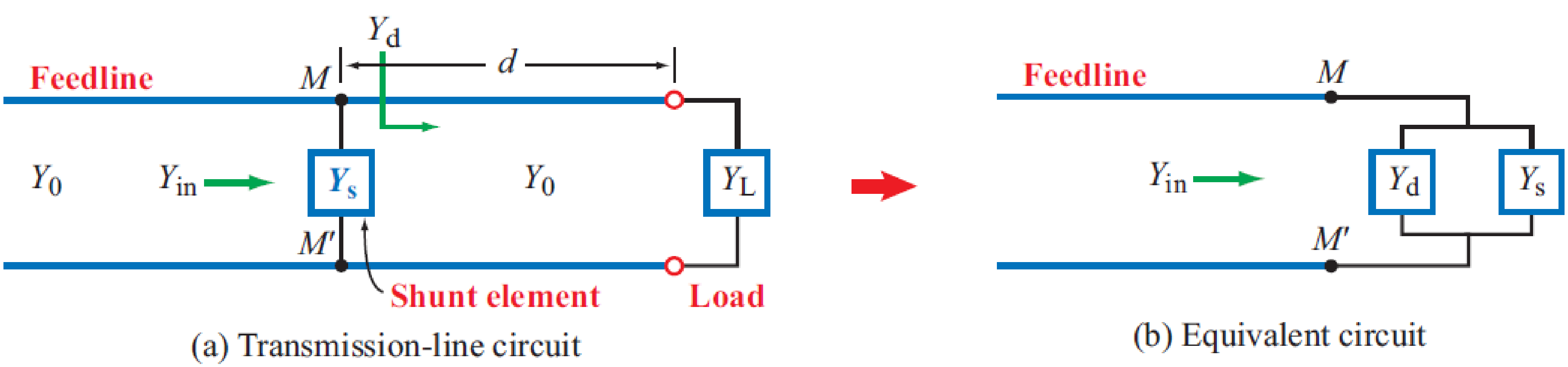


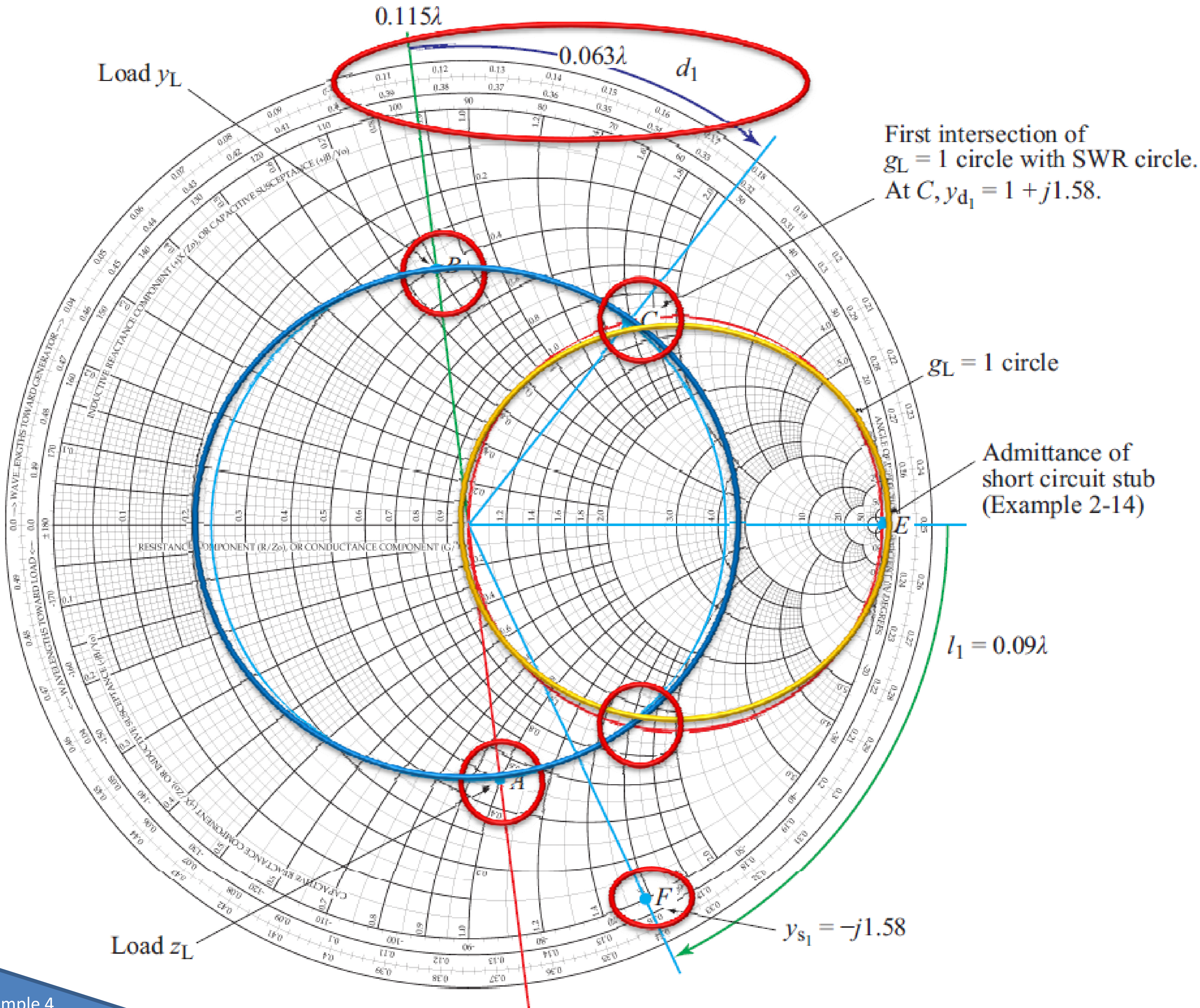
Figure 2-34: Inserting a reactive element with admittance Y_s at MM' modifies Y_d to Y_{in}.

Example of a Matching Network

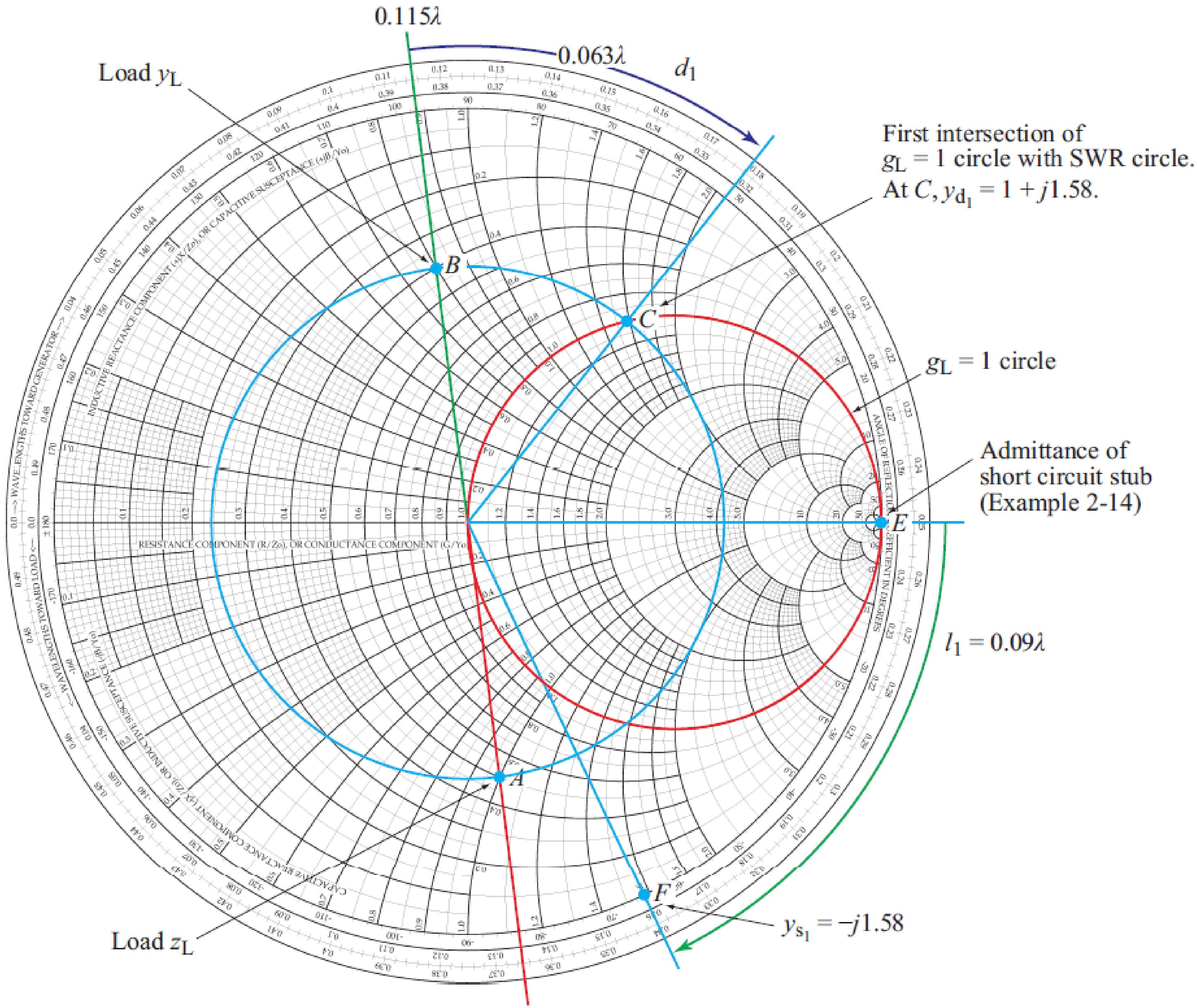
A load impedance $Z_L = 25 - j50\ \Omega$ is connected to a 50-Ω transmission line. Insert a shunt element to eliminate reflections towards the sending end of the line. Specify the insert location d (in wavelengths), the type of element, and its value, given that $f = 100$ MHz.

$$z_L = \frac{Z_L}{Z_0} = \frac{25 - j50}{50} = 0.5 - j1$$

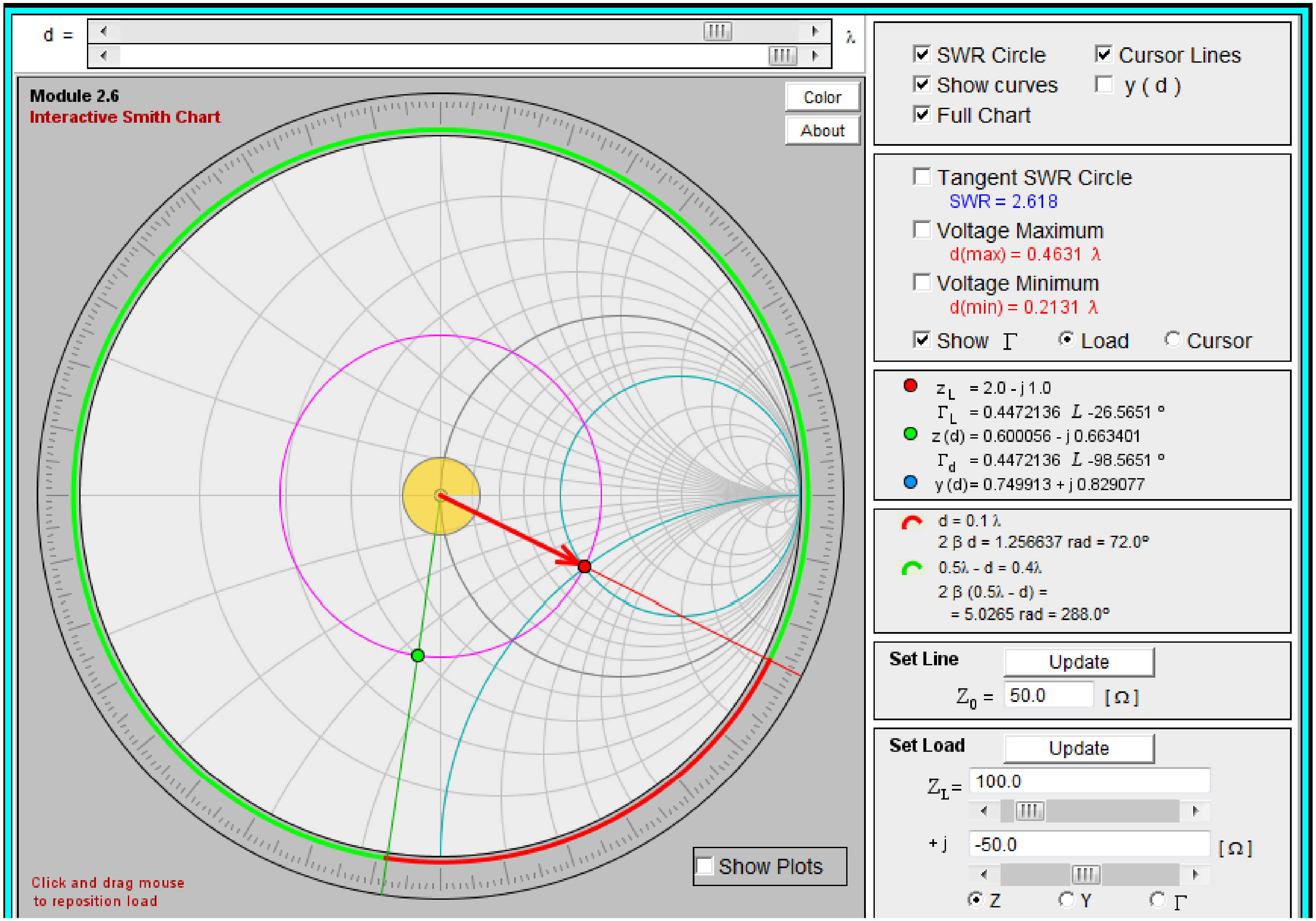
$$y_L = 0.4 + j0.8$$


0.115λ
0.063λ
d1
Load yL
First intersection of
gL = 1 circle with SWR circle.
At C, yd1 = 1 + j1.58.
gL = 1 circle
Admittance of
short circuit stub
(Example 2-14)
l1 = 0.09λ
ys1 = −j1.58
Load zL
B
C
A
E
F

Example 4


0.115λ
0.063λ
d1
Load yL
B
C
First intersection of
gL = 1 circle with SWR circle.
At C, yd1 = 1 + j1.58.
gL = 1 circle
Admittance of
short circuit stub
(Example 2-14)
E
l1 = 0.09λ
A
F
ys1 = −j1.58
Load zL
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES

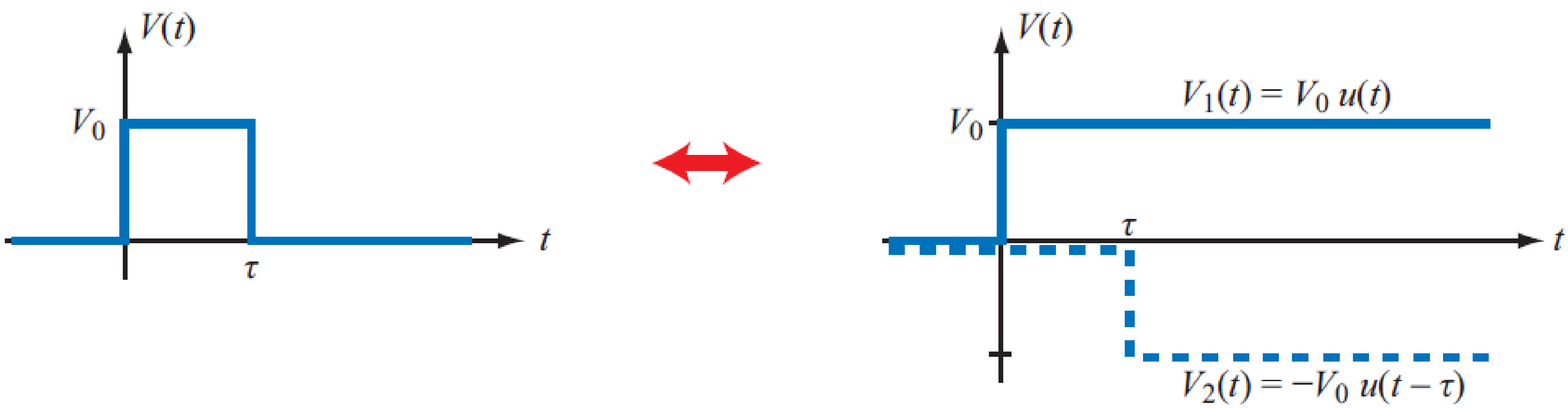
Use the CD Smith Chart



Transient of Transmission Line

Transients

*The **transient response** of a voltage pulse on a transmission line is a time record of its back and forth travel between the sending and receiving ends of the line, taking into account all the multiple reflections (echoes) at both ends.*

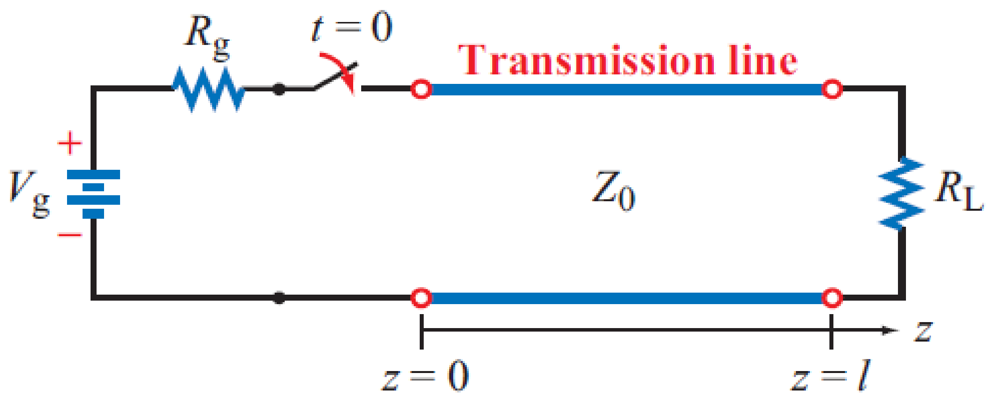


(a) Pulse of duration τ

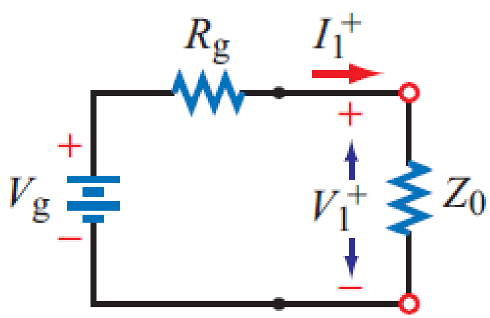
(b) $V(t) = V_1(t) + V_2(t)$

Rectangular pulse is equivalent to the sum of two step functions

Transient Response



(a) Transmission-line circuit



(b) Equivalent circuit at $t = 0^+$

Initial current and voltage

$$I_1^+ = \frac{V_g}{R_g + Z_0},$$

$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$

Reflection at the load

$$V_1^- = \Gamma_L V_1^+,$$

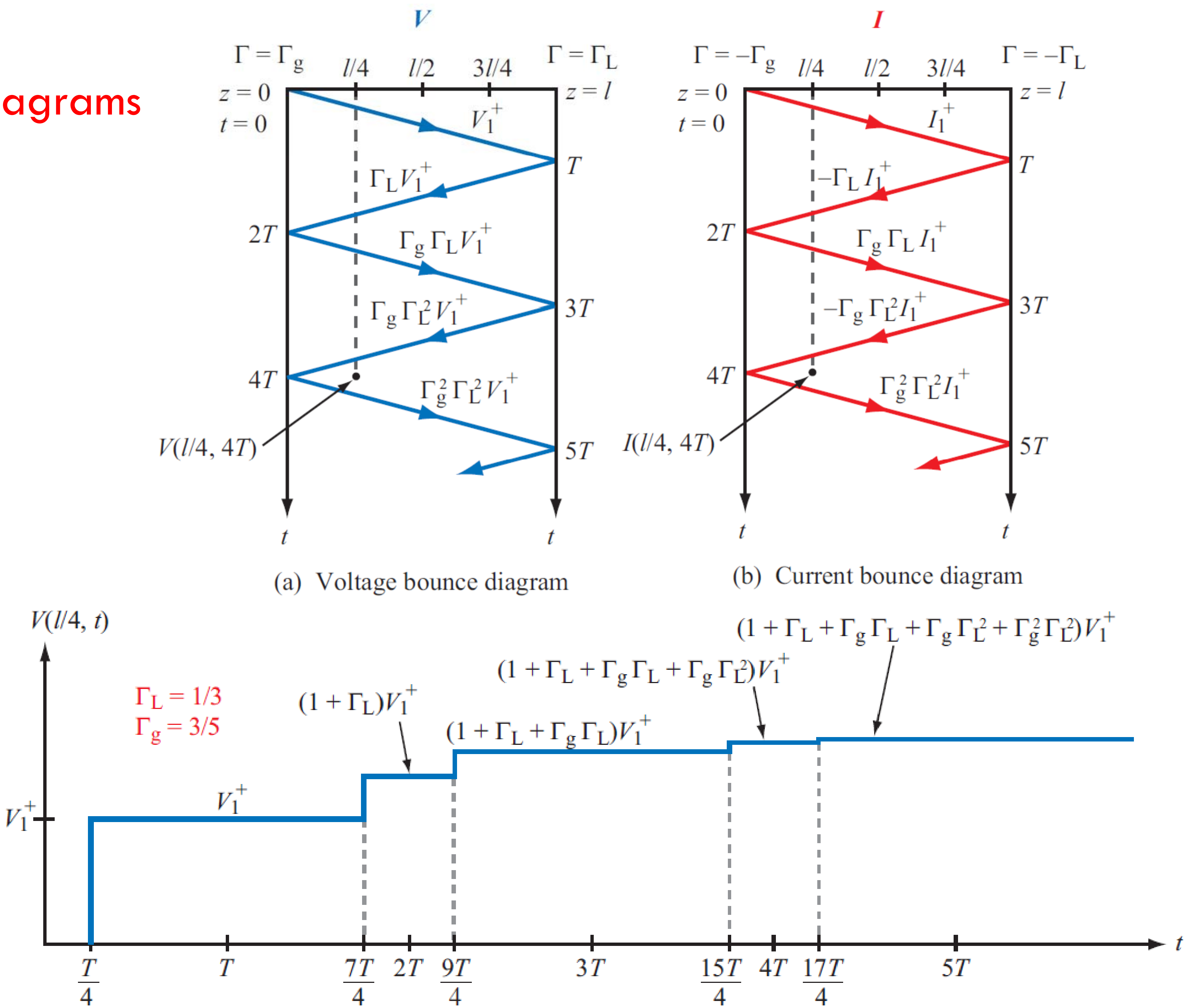
Load reflection coefficient $\Gamma_L = \dfrac{R_L - Z_0}{R_L + Z_0}$

Second transient

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

Generator reflection coefficient $\Gamma_g = \dfrac{R_g - Z_0}{R_g + Z_0}$

Bounce Diagrams



(a) Voltage bounce diagram

(b) Current bounce diagram

(c) Voltage versus time at $z = l/4$